International Journal of Engineering Research & Management Technology ISSN: 2348-4039
January-2016 Volume 3, Issue-1
Email: editor@ijermt.org www.ijermt.org

Classify Stage Inventory Model with Decreasing Demand and Unpredictable Deterioration

Gajendra Singh, Deepika Raj
Research Scholar
Sai Nath University
Ranchi

ABSTRACT:
Business inventory is spread profusely throughout the economy, predominately in manufacturing, wholesaling, and retailing organizations. Manufacturing organizations always transform raw materials into final products, while service organizations only use materials as an adjunct to their final products. The effective management of inventory is crucial to the performance of many organizations. It can have serious implications for the finance, production, and marketing functions of any organizations. Finance is influenced through liquidity and return on investment, production through efficiency and cost of operations, and marketing through sales and customer relations., In this paper, the devoted to the development of an order level inventory model with decreasing demand and unpredictable deterioration. The demand rate is linearly decreasing function of time. The deterioration rate has been taken of the form $\theta(t) = \alpha\beta t^{\beta-1}$. The production rate has been assumed proportional to demand rate.

KEY WORDS:

INTRODUCTION:

Many of the physical goods undergo decay or deterioration over time. Commodities such as fruits, vegetables, and foodstuffs are subject to direct spoilage during storage period. The highly volatile liquids such as gasoline, alcohol, and turpentine undergo physical depletion over time through the process of evaporation. The electronic goods, radioactive substances, photographic film, and grain deteriorate through a gradual loss of potential or utility with passage of time. The deteriorated items repairing is a major problem in the supply chain of most of the business organizations. Traditionally, management devoted more time to the expenditure of moneys for personnel, plant and equipment than for materials. Designing, financing, manufacturing of materials is crucial to the performance of many organizations therefore they are the major organizational functions. Materials were thought of as cheap, readily available, and infinitely plentiful. The realities of the market place have changed this myopic view and have added materials to the list of major organizational functions. The control and maintenance of inventory is a problem common to all organizations in any sectors of the economy. The problem of inventory is a problem common. The problem of inventory does not confine themselves to profit-making institutions but like wise are encountered by social and non-profit institutions. Inventories are common to farms, manufactures, wholesalers, retailers, hospitals, churches, prisons, zoos, universities and national, state and local governments. Indeed, inventories are also relevant to the family unit in relation to food, clothing, medicines, toiletries and so forth. Most of the earlier inventory models consider that demand rate is constant. This is a feature of static environment, while in today's dynamic environment most of the things are not constant. In real life situations generally demand for items increases with time. Most of the companies are working towards increasing demand of their items with time.

REVIEW OF LITERATURE:

In a few of these works, deterioration rate is not constant. For example, Chen [J.M. Chen, 1998] proposed an inflationary model with time proportional demand andWeibull distribution for deteriorating items using dynamic programming. Sana [2011] proposed a model which deals with a stochastic economic order quantity (EOQ) model over a finite time horizon where uniform demand over the replenishment period is price dependent. The selling price is assumed to be a random variable that follows a probability density function. Moreover, the article suggests a new function regarding price-dependent demand. Sana [2010] also presented another EOQ model over an infinite time horizon for perishable items where demand is price dependent and partial backorder is permitted. Based on the partial backlogging and lost sale cases, the author develops the criterion for the optimal solution for the replenishment schedule and proves that the optimal ordering policy is unique. Balkhi [2004] presented a production lot-size inventory model where the production, demand, and deterioration rates are known, continuous, and differentiable functions of time. Shortages are allowed, but only a fraction of the stockout is backordered, and the rest is lost. De and Sana [2013] developed a research that deals with a backorder EOQ model with promotional index for fuzzy decision variables. Here, a profit function is developed, where the function itself is the function of the power of promotional index (PI) and the order quantity; the shortage quantity and the PI are the decision variables. The demand rate is operationally related to PI variables and the model has been split into two types for the multiplication and addition operations. Lo et al. [2009] introduced an integrated production-inventory model with imperfect production processes and Weibull distribution deterioration under inflation from the perspectives of both the manufacturer and the retailer. Sana [2012] developed an EOQ model to determine the retailer's optimal order quantity for similar products. It is assumed that the amount of display space is limited and the demand of the products depends on the display stock level and the initiatives of sales staff where more stock of one product makes a negative impression of the other product. Roy Chowdhury et al. [2014] presented an inventory model for perishable items with stock and advertisement sensitive demand. Singh and Pattnayak [2014] developed a two-warehouse inventory model for deteriorating items with linear demand under conditionally permissible delay in payment. Sivashankari and Panayappan [2014] studied a production-inventory model with reworking of imperfect production, scrap, and shortages. Effect of inflation and time value of money in inventory problems is well established. The initial attempt in this direction was made by Buzacott [1975]. He dealt with an EOQ model under inflation subject to different types of pricing policies. Soleimani-Amiri et al. [2014] compared two methods for calculating the optimal total cost in an inventory model with time value of money and inflation for deteriorating items. Hou [2006] developed an inflation model for deteriorating items with stock-dependent consumption rate and completely backordered shortages by assuming a constant length of replenishment cycles and a constant fraction of the shortage length with respect to the cycle. Chern et al. [2008] proposed partial backlogging inventory lot-size models for deteriorating items with fluctuating demand under inflation.

ASSUMPTIONS AND NOTATIONS:

The present inventory model has been developed under the following assumption and notations.

1. The demand rate $d(t)$ =a-bt, a and b are constants and $a >> b$, is a decreasing function of time.
2. The replenishment rate is $r(t)$= $\gamma\, d(t)$, where $\gamma \succ 1$ is a constant.
3. The deterioration rate is $\theta(t) = \alpha\beta t^{\beta-1}, \sigma < \propto < 1$, $t < 0$, $\beta \geq 1$.
4. The lead time is zero and shortages are not allowed
5. The deteriorated items are neither replaced nor repaired during the cycle time.
6. C is unit cost, C_1 is the unit holding cost per time , A is the ordering cost which is fixed, T is the cycle time and TC is the total cost per unit time.

MATHEMATICAL MODEL:

Let $q(t)$ be the inventory level at any time t, $0 \le t \le T$. The governing differential equations of inventory system in the interval $[0, T]$ are given by.

$$\frac{dq}{dt} + \theta(t)q = r(t) - d(t), \qquad 0 \le t \le t_1 \qquad \ldots(1.1)$$

and
$$\frac{dq}{dt} + \theta(t)q = -d(t), \qquad t_1 \le t \le T \qquad \ldots(1.2)$$

The stock level is zero initially at the beginning of production time i.e. $q(0) = 0$. It continues up to the time $t = t_1$ and stock with stock level S. Finally, the stock level reduces to zero at $t=T$ due to demand and deterioration. This completes one cycle.

The objective is to obtain the optimal values of S and $T\,C$ subject to the decision unpredictable's t_1 and T. By the substitution of values of $d(t)$ and $\theta(t)$ the equation (1.1) and (1.2) become

$$\frac{dq}{dt} + \alpha\beta t^{\beta-1} q = (y-1)(a-bt), \qquad 0 \le t \le t_1 \qquad \ldots(1.3)$$

and
$$\frac{dq}{dt} + \alpha\beta t^{\beta-1} q = -(a-bt), \qquad t_1 \le t \le T \qquad \ldots(1.4)$$

The solution of equation (1.3) with the initial condition is given by

$$q = (\gamma - 1)\left[at - \frac{bt^2}{2} + \frac{a\alpha t^{\beta+1}}{\beta+1} - \frac{\alpha b t^{\beta+2}}{\beta+2} - \right]e^{-\propto t^{\beta}}. \qquad \ldots(1.5)$$

Similarly the solution of equation (1.4) is given by.

$$qe^{\propto t^{\beta}} = \left(at - \frac{bt^2}{2} + \frac{a\alpha t^{\beta+1}}{\beta+2}\right) + c_2.$$

The condition $q(t_1) = S,$ we get.

$$Se^{\propto t_1^{\beta}} = \left(at, - \frac{bt_1^{\,2}}{2} + \frac{a \propto t_1^{\,\beta+1}}{\beta+1} - \frac{\propto bt_1^{\,\beta+2}}{\beta+2}\right) + c_2$$

This gives

$$c_2 = S\left(1 - \alpha t_1^{\,\beta}\right) + \left(at_1 - \frac{bt_1^{\,2}}{2} + \frac{a\alpha t_1}{\beta+1} - \frac{\alpha b t_1}{\beta+2}\right)$$

$$\therefore q(t) = -\left(at - \frac{bt^2}{2} + \frac{a\alpha t^{\beta+1}{}_1}{\beta+1} - \frac{\alpha bt}{\beta+2}\right)$$

$$+ S\left(1 - \alpha t_1^{\beta}\right) + \left(at_1 - \frac{bt_1^2}{2} + \frac{a\alpha t_1^{\beta+1}}{\beta+1} - \frac{b\alpha t_1^{\beta+2}}{\beta+2}\right). \qquad \ldots(1.6)$$

Using the condition $q(T) = o$ in (1.6) we get

$$S = \frac{1}{\left(1 - \alpha t_1^{\,\beta}\right)}\left[a\left(T - t_1 \frac{\alpha T^{\beta+1}}{\beta+1} - \frac{\alpha t_1^{\,\beta+1}}{\beta+1}\right)\right]$$

$$-b\left(\frac{T^2 - t_1^{\,2}}{2} - \alpha\left(\frac{T^{\beta+2} - t_1^{\,\beta+2}}{\beta+2}\right)\right). \qquad \ldots(1.7)$$

Now the average holding cost is given by

$$HC=\frac{C_1}{T}\left[\int_0^{t_1} qdt+\int^{t_1} qdt\right]$$

$$=\frac{C_1}{T}\left[\int_0^{T_1}(\gamma-1)\left[AT-\frac{bt^2}{2}+\frac{a\alpha t^{\beta+1}}{(\beta+1)}-\frac{\alpha bt^{\beta+2}}{\beta+2}\right]\left[1-\alpha t^{\beta}\right]dt\right.$$

$$+\int_{t_1}^{T_1}[S\left(1-\alpha t_1^{\beta}\right)+a\left(t_1-t\right)-\frac{b}{2}\left(t_1^2-t^2\right)$$

$$+\frac{a\alpha}{\beta+1}\left(t_1^{\beta+1}-t^{\beta+1}\right)-\frac{\alpha b}{\beta+2}\left(t_1^{\beta+2}-t_{dt}^{\beta+2}\right)]dt$$

$$=\frac{C_1}{T}\left[\frac{a}{2}\left\{(\gamma-1)t_1^2-\frac{2\alpha\beta(\gamma-1)}{(\beta+1)(\beta+2)}-\frac{(\gamma-1)\alpha^2 t_1^{2\beta+2}}{(\beta+1)^2}\right.\right.$$

$$+2t_1T-T^2-t_1^2+\frac{2\alpha}{\beta+1}\left(t_1^{\beta+1}T-\frac{\beta+2}{(\beta+2)}-\left(\frac{\beta+1}{\beta+2}\right)t_1^{\beta+2}\right)\}$$

$$-\frac{b}{2}\left\{\frac{(\gamma-1)t_1^3}{3}-\frac{\alpha\beta(\gamma-1)t_1^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{2\alpha^2(\gamma-1)t_1^{2\beta+3}}{(\beta+2)(\beta+3)}\right.$$

$$+t_1^2T-\frac{T^3}{3}-\frac{2}{3}t^3-\frac{2\alpha}{(\beta+2)}\left(t_1^{\beta+2}T-\frac{T^{\beta+3}}{\beta+3}-\frac{(\beta+2)}{(\beta+3)}t_1^{\beta+3}\right)+S\left(1-\propto t_1^{\beta}\right)\left(T-t_1\right)]. \quad \ldots(1.8)$$

The average deterioration cost per unit time is given by

$$DC=\frac{C}{T}\left[\int_0^{t_1} r(t)dt-\int_0^{T} d(t)dt\right]$$

$$=\frac{C}{T}\left[\gamma\int_0^{t_1}(a-bt)dt-\int_0^{T}(a-bt)dt\right]$$

$$=\frac{C}{T}\left[a(\gamma t_1-T)-\frac{b}{2}\left(\gamma t_1^2-T^2\right)\right]. \quad \ldots(1.9)$$

The average cost of the inventory system is given by

$$TC=HC+DC+A. \quad \ldots(1.10)$$

$$=\frac{C_1}{T}[(\gamma-1)\left[\frac{at_1^2}{2}-\frac{bt_1^3}{6}-\frac{a\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{b\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}\right.$$

$$-\frac{a\alpha^2 t_1^{2\beta+2}}{2(\beta+1)^2}+\frac{b\alpha^2 t_1^{2\beta+3}}{(\beta+2)(\beta+3)}]+S\left(1-\alpha t_1^{\beta}\right)\left(T-t_1\right)+a\left[t_1T-\frac{1}{2}\left(T^2+t_1^2\right)\right]$$

$$-\frac{b}{2}\left[t_1^2T-\frac{T^3}{3}-\frac{2}{3}T_1^3\right]+\frac{a\alpha}{(\beta+1)}\left(t_1^{\beta+1}T-\frac{T^{\beta+2}}{(\beta+2)}-t_1^{\beta+2}\frac{\beta+1}{\beta+1}\right)$$

$$\frac{\alpha b}{\beta+2}\left(t_1^{\beta+2}T-\frac{T^{\beta+3}}{\beta+3}-\frac{(\beta+2)^{\beta+3}}{(\beta+3)}t_1^{\beta+3}\right)]$$

$$=\frac{C_1}{T}\left[\frac{a}{2}\left\{(\gamma-1)t_1^2-\frac{2\alpha\beta(\gamma-1)}{(\beta+1)(\beta+2)}-\frac{(\gamma-1)\alpha^2 t_1^{2\beta+2}}{(\beta+1)^2}\right.\right.$$

$$+2t_1T-T^2-t_1^2+\frac{2\alpha}{\beta+1}\left(t_1^{\beta+1}T-\frac{\beta+2}{(\beta+2)}-\left(\frac{\beta+1}{\beta+2}\right)t_1^{\beta+2}\right)\}$$
$$-\frac{b}{2}\left\{\frac{(\gamma-1)t_1^3}{3}-\frac{\alpha\beta(\gamma-1)t_1^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{2\alpha^2(\gamma-1)t_1^{2\beta+3}}{(\beta+2)(\beta+3)}\right.$$
$$+t_1^2T-\frac{T^3}{3}-\frac{2}{3}t^3-\frac{2\alpha}{(\beta+2)}\left(t_1^{\beta+2}T-\frac{T^{\beta+3}}{\beta+3}-\frac{(\beta+2)}{(\beta+3)}t_1^{\beta+3}\right)+S\left(1-\alpha t_1^{\beta}\right)(T-t_1)].\qquad\ldots(1.11)$$

Now for the optimal values of TC, we have to differentiate TC w.r.t. t_1 and T partially and put them equal to zero. Thus we have

$$\frac{1}{T}\left[\frac{a}{2}\left\{2(\gamma-1)t_1-\frac{2(\gamma-1)\alpha^2t_1^{2\beta+1}}{(\beta+1)}\right.\right.$$
$$+2T-2t_1+2\alpha\left(t_1^{\beta}T-t_1^{\beta+1}\right)\}$$
$$-\frac{b}{2}\left\{(\gamma-1)t_1^2-\frac{\alpha\beta(\gamma-1)t_1^{\beta+2}}{(\beta+2)}-\frac{2\alpha^2(\gamma-1)(2\beta+3)t_1^{2\beta+2}}{(\beta+2)(\beta+3)}\right.$$
$$+2t_1T-2t^2-2\alpha\left(t_1^{\beta+1}T-t_1^{\beta+2}\right)+S\left(-1-\alpha\beta t_1^{\beta-1}T+\alpha(\beta+1)t_1^{\beta}\right)]=0,\qquad\ldots(1.12)$$

$$\left[-\frac{1}{T^2}\frac{a}{2}\left\{(\gamma-1)t_1^2-\frac{2\alpha\beta(\gamma-1)}{(\beta+1)(\beta+2)}-\frac{(\gamma-1)\alpha^2t_1^{2\beta+2}}{(\beta+1)^2}\right.\right.$$
$$-1+\frac{1}{T^2}t_1^2+\frac{2\alpha}{\beta+1}\left(\frac{\beta+2}{T^2(\beta+2)}+\frac{1}{T^2}\left(\frac{\beta+1}{\beta+2}\right)t_1^{\beta+2}\right)\}$$
$$-\frac{b}{2}\left\{-\frac{(\gamma-1)t_1^3}{3T^2}+\frac{\alpha\beta(\gamma-1)t_1^{\beta+3}}{(\beta+2)(\beta+3)T^2}+\frac{2\alpha^2(\gamma-1)t_1^{2\beta+3}}{(\beta+2)(\beta+3)T^2}\right.$$
$$-\frac{2T}{3}+\frac{2}{3T^2}t^3-\frac{2\alpha}{(\beta+2)}\left(-\frac{(\beta+2)T^{\beta+1}}{\beta+3}+\frac{(\beta+2)}{(\beta+3)T^2}t_1^{\beta+3}\right)+S\left(1-\alpha t_1^{\beta}\right)\frac{t_1}{T^2}]=0.\qquad\ldots(1.13)$$

The values of t_1 and T obtained from (1.12) and (1.13) will be optimal provided they satisfy the inequalities

$$\frac{\partial^2TC}{\partial t_1^2}>0,\qquad\frac{\partial^2TC}{\partial T^2}>0$$

and $$\frac{\partial^2TC}{\partial t_1^2}.\frac{\partial^2TC}{\partial T^2}-\left(\frac{\partial^2TC}{\partial t_1\partial T}\right)^2>0.$$

These values of t_1 and T will minimize the total average cost. These values can be obtained with the help of suitable numerical method.

CONCLUSIONS:

At this time, order level inventory model with linearly decreasing demand and unpredictable deterioration rate has been developed. For β=2, the model reduces to that of linear deterioration rate and for β=1, the model reduces to that of constant deterioration rate. The model further can be modified for the cases of partial backlogging and fully backlogging. The effect of inflation can also be studied on the present model. The present model can also be extended for other forms of demand rates. The concept of life time can also be included in the further extension of the present model.

REFERENCES:

1. S. K. De and S. S. Sana, "Fuzzy order quantity inventory model with fuzzy shortage quantity and fuzzy promotional index," Economic Modelling, vol. 31, no. 1, pp. 351–358, 2013.
2. R. Roy Chowdhury, S. K. Ghosh, and K. S. Chaudhuri, "An inventory model for perishable items with stock and advertisement sensitive demand," International Journal of Management Science and Engineering Management, vol. 9, no. 3, pp. 169–177, 2014.
3. C. K. Sivashankari and S. Panayappan, "Production inventory model with reworking of imperfect production, scrap and shortages," International Journal of Management Science and Engineering Management, vol. 9, no. 1, pp. 9–20, 2014.
4. K.K.Bansal,U.P.Singh,"Develop an Inventory Model with Time Dependent Deterioration, and Demand Rate under the Condition of Permissible Delay" International Journal of Engineering Research & Management Technology, vol.1 Issue 5
5. K.K.Bansal,Adarsh Kumar," A Deterministic Inventory Model for a Deteriorating Item Is Explored In an Inflationary Environment for an Infinite Planning Horizon" International Journal of Education and Science Research Review, vol.1 Issue 4
6. K.K.Bansal,Navin Ahlawat,"Optimal Ordering Decision for Deteriorated Items with Expiration Date and Uncertain Lead Time" International Journal of Management Research
7. and Review, Vol.2 Issue 6
8. K.K.Bansal," Order Level Inventory Model With Decreasing Demand And Variable Deterioration" International Journal of Engineering & Science Research, Vol.2 Issue 9
9. Mirzazadeh, "Inventory management under stochastic conditions with multiple objectives," Artificial Intelligence Research, vol. 2, pp. 16–26, 2013.
10. M. Ameli, A. Mirzazadeh, and M. A. Shirazi, "Economic order quantity model with imperfect items under fuzzy inflationary conditions," Trends in Applied Sciences Research, vol. 6, no. 3, pp. 294–303, 2011
11. D. K. Jana, B. Das, and T. K. Roy, "A partial backlogging inventory model for deteriorating item under fuzzy inflation and discounting over random planning horizon: a fuzzy genetic algorithm approach," Advances in Operations Research, vol. 2013, Article ID 973125, 13 pages, 2013.
12. K. Neetu and A. Tomer, "Deteriorating inventory model under variable inflation when supplier credits linked to order quantity," Procedia Engineering, vol. 38, pp. 1241–1263, 2012.
13. K. L. Hou and L. C. Lin, "Optimal pricing and ordering policies for deteriorating items with multivariate demand under trade credit and inflation," OPSEARCH, vol. 50, no. 3, pp. 404–417, 2013.
14. S. M. Mousavi, V. Hajipour, S. T. Niaki, and N. Alikar, "Optimizing multi-item multi-period inventory control system with discounted cash flow and inflation: two calibrated meta-heuristic algorithms," Applied Mathematical Modelling, vol. 37, no. 4, pp. 2241–2256, 2013.
15. M. Ghoreishi, A. Arshsadi Khamseh, and A. Mirzazadeh, "Joint optimal pricing and inventory control for deteriorating items under inflation and customer returns," Journal of Industrial Engineering, vol. 2013, Article ID 709083, 7 pages, 2013.
16. Z. T. Balkhi, "An optimal solution of a general lot size inventory model with deteriorated and imperfect products, taking into account inflation and time value of money," International Journal of Systems Science, vol. 35, no. 2, pp. 87–96, 2004.
17. S. S. Sana, "Optimal selling price and lotsize with time varying deterioration and partial backlogging," Applied Mathematics and Computation, vol. 217, no. 1, pp. 185–194, 2010.
18. R. Roy Chowdhury, S. K. Ghosh, and K. S. Chaudhuri, "An inventory model for perishable items with stock and advertisement sensitive demand," International Journal of Management Science and Engineering Management, vol. 9, no. 3, pp. 169–177, 2014.
19. T. Singh and H. Pattnayak, "Two-warehouse inventory model for deteriorating items with linear demand under conditionally permissible delay in payment," International Journal of Management Science and Engineering Management, vol. 9, no. 2, pp. 104– 113, 2014.
20. K. Sivashankari and S. Panayappan, "Production inventory model with reworking of imperfect production, scrap and shortages," International Journal of Management Science and Engineering Management, vol. 9, no. 1, pp. 9–20, 2014
21. M. Soleimani-Amiri, S. Nodoust, A. Mirzazadeh, and M. Mohammadi, "The average annual cost and discounted cost mathematical modeling methods in the inflationary inventory systems—a comparison analysis," SOP Transactions on Applied Mathematics, vol. 1, no. 1, pp. 31–41, 2014.